Hydromagnetic Instability Criteria for Stratified Viscoelastic Dusty Finitely Conducting Plasmas

M. F. El-Sayed

Department of Mathematics and Computer Science, Faculty of Science, U. A. E. University, P. O. Box 17551, Al Ain, United Arab Emirates; and Department of Mathematics, Faculty of Education, Ain Shams University, Roxy, Heliopolis, Cairo, Egypt

Reprint requests to Dr. M. F. El-Sayed, E-mail: m.elsayed@uaeu.ac.ae

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The instability of a stratified, viscoelastic (Walters B' or Rivlin-Ericksen) magnetoplasma, including the effects of finite resistivity and suspended particles, is investigated using linear analysis. The horizontal applied magnetic field and the viscosity, as well as the viscoelasticity of the medium are assumed to be variable. The dispersion relation, which is obtained for the general case on employing boundary conditions appropriate to the case of two free boundaries, is then specialized for the two models. The hydromagnetic instability conditions are obtained and discussed analytically, and the results are numerically confirmed. The variation of the growth rate of the unstable modes with the different parameters has been evaluated analytically. All the physical parameters are found to have stabilizing as well as destabilizing effects on the considered system. For the Walters B' viscoelastic model it was found that the kinematic viscoelasticity, fluid resistivity, and stratification parameter have a stabilizing effect, while the mass concentration (or relaxation frequency) of the suspended particles, kinematic viscosity, and Alfvén velocity have a destabilizing effect on the considered system. Also, for the case of the Rivlin-Ericksen viscoelastic model we found that the mass concentration of the suspended particles, Alfvén velocity, and kinematic viscosity have a stabilizing effect, while both the finite resistivity and stratification parameter have a destabilizing effect; the relaxation frequency of the suspended particles has no effect on the stability of the system. The case of a dusty plasma with infinite conductivity and presence (or absence) of a magnetic field is also considered. Its stability conditions are obtained, from which it is concluded that the presence of dust always reduces the growth rate of the unstable Rayleigh-Taylor perturbations. The limiting case of a viscid (and inviscid) finitely conducting dusty plasma is considered, and the stability conditions are discussed, from which we found that the magnetic field has a stabilizing effect in the absence of both viscosity and finite resistivity, the stability of the system ocurrs for values of the Alfvén velocity greater than a critical value. — PACS: 47.20.-k; 47.50+d; 47.65.+a.

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1. Introduction

The Rayleigh-Taylor instability occurs when a heavier fluid rests on a lighter one. Then the havier fluid forms spikes moving into the lighter one, and the lighter fluid forms bubbles penetrating the heavier fluid. Therefore a mixing zone developes, which grows with time. A comprehensive account of the Rayleigh-Taylor instability under various assumptions on the viscosity and magnetic field has been given by Chandrasekhar [1].

The study of viscoelastic fluids has become important in recent years because of their many applications in petroleum drilling, manufacturing of foods and paper, etc. [2 – 4]. Walters [5], and Beard and Walters [6] have deduced the equations for the boundary flow of a viscoelastic fluid which they have called liquid B' when it has a very short memory. Raptis et al. [7 – 9] have studied the free convection and mass transfer of a viscous and viscoelastic fluid passing a vertical wall. Chakraborty and Sengupta [10] have studied the flow of an unsteady viscoelastic liquid B' through two porous concentric non-conducting infinite circular cylinders rotating with different angular velocities in the presence of a uniform axial magnetic field. Sharma and Kumar [11] have studied the steady flow and heat transfer of a liquid B' through a porous pipe of uniform circular cross-section. They have also studied the unsteady flow of a liquid B' down an open inclined channel under gravity. For recent works about the subject see the papers of Othman [12], El-Sayed [13 – 15], Sharma and his colaborators [16 – 18], and Kumar [19]. There are also many viscoelastic fluids that can not be characterized by Oldroyd's constitutive relation. One such class are the Rivlin-Ericksen viscoelastic fluids [14]. Many workers have studied the Rivlin-Ericksen viscoelastic fluids, see for example the recent investigations of Sharma and Kumar [16], Özer and Suhubi [20], Sharma et al. [17], and El-Sayed [14], among others.

Dusty plasmas occur often in nature. They are also present in many laboratory devices [21]. The importance of dusty plasmas in star formation and other astrophysical problems has recently attracted the scientists. Interstellar and circumstellar clouds are full of dust particles. Dust grains exist also in comets, planetary rings, asteroids, and the magnetosphere, as well as in the lower ionosphere [22]. The effect of dust on the hydromagnetic stability of various fluids might be of chemical engineering importance. The problem of dusty gas in magnetohydrodynamics has been discussed by various authors. Sharma et al. [23] have studied the effect of suspended particles on the onset of Bènard convection. Suspended particles were found to stabilize the Bènard convection. Sharma and Sharma [24] have also analyzed the Rayleigh-Taylor instability of a medium consisting of two superposed fluids with suspended particles. They showed that the system remains uninfluenced by the presence of suspended particles. Chhajlani et al. [25] have incorporated the effect of suspended particles on the Rayleigh-Taylor instability problem of a viscous plasma having an exponentially varying density distribution in the presence of a variable horizontal magnetic field, considering the medium to be infinitely conducting. The growth rate of the unstable Rayleigh-Taylor modes has been evaluated analytically in order to examine the influence of both the suspended particles and the viscosity. For recent works concerning dusty plasmas see the papers of Shukla and Rahman [26], Birk et al. [27], Birk [28], and Maury and Glowinski [29]. For an excellent review see Verheest [30].

The equilibrium states of electrically conducting fluids or plasmas have been a subject of intense study for a long time, motivated in particular by the interest in controlled thermonuclear fusion, as well as that in space and astrophysical phenomena such as plasma loops in the solar corona [31 – 33]. The approximation of an ideally conducting plasma is valid for an astrophysical plasma, but it is often a poor approximation for laboratory plasmas, and hence the finite resistivity needs to be incoporated in a more realistic approach [34]. It has been demonstrated by Furth et al. [35], Jukes [36], and many others that the inclusion of finite resistivity modifies the Rayleigh-Taylor problem and makes possible new unstable modes. Zadoff and Begun [37] have treated the case of two incompressible fluids separated by a horizontal boundary in the presence of a uniform horizontal magnetic field. They have discussed the effects of finite resistivity and viscosity of the medium on the growth rate of Rayleigh-Taylor modes, and have shown that a finite resistivity does not affect the growth rate of unstable modes when the wave vector is perpendicular to the magnetic field, but that it does increase the growth rate when the wave vector is parallel to the magnetic field. Sundram [38] has considered gravitational instability of a fluid finite resistivity and concluded that the density stratification is unstable for all wavenumbers. Sanghvi and Chhajlani [39] have incorporated the finite resistivity effect on the Rayleigh-Taylor configuration of a stratified plasma in the presence of suspended particles, and found that the particles have stabilizing as well as destabilizing influences on the system under certain conditions.

In this regard and in view of the practical importance of including the effect of a magnetic field [40] and fluid density stratification [41] to the hydrodynamic stability problems, it is of interest to investigate the simultaneous inclusion of finite resistivity and suspended particles in the analysis of the Rayleigh-Taylor instability of a stratified magnetized viscoelastic Walters B' or Rivlin-Ericksen fluid acted upon by a horizontal magnetic field, and discussed various implifications of the finite resistivity corrections, kinematic viscosity, kinematic viscoelasticity, and suspended particles. To the best of my knowledge this problem has not been investigated yet. The main topic of this paper is to investigate the effects of the various above mentioned physical parameters on the hydromagnetic instability of two viscoelastic models through dusty plasmas.

2. Basic and Perturbation Equations

Consider a homogeneous magnetized (Walters B' or Rivlin-Erichsen) viscoelastic medium of a gas with suspended particles, which is infinite along the x- and y-directions, and bounded by two free surfaces at $z = 0$ and $z = d$. Let $\boldsymbol{u}$, $\boldsymbol{v}$, N, and ρ denote the gas velocity, the velocity of the particles, the number density of the particles, and the gas density, respectively. The particles are assumed to be nonconducting and the gas is considered finitely conducting and viscoelastic. If we assume uniform particle size, spherical shape and small relative velocities between the two phases, then the net effect of the particles on the gas is equivalent to an external force term per unit volume $KN(\boldsymbol{v}-\boldsymbol{u})$, where $K = 6\pi\rho\nu r$ (Stokes'drag formula), and r and ν denote the particle radius, and the kinematic viscosity of the gas, respectively. Hence, the relevant equation of motion for the hydromagnetic gas-particle viscoelastic medium is

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla p + \boldsymbol{g}\rho + \rho\left(\nu \mp \nu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{u} + KN(\boldsymbol{v}-\boldsymbol{u}) + \frac{\mu_e}{4\pi}(\nabla\times\boldsymbol{H})\times\boldsymbol{H} + \left(\frac{\partial w}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{u}}{\partial z}\right)\left(\frac{\mathrm{d}\mu}{\mathrm{d}z} \mp \frac{\partial}{\partial t}\frac{\mathrm{d}\mu'}{\mathrm{d}z}\right), \tag{1}$$

where μ, μ', $\nu(=\mu/\rho)$, and $\nu'(=\mu'/\rho)$ stand for the viscosity, viscoelasticity, kinematic viscosity, and kinematic viscoelasticity, respectively, while p, μ_e, $\boldsymbol{g}(0,0,-g)$, and $\boldsymbol{H}_0(H(z),0,0)$ denote the pressure, the magnetic permeability of the gas, the gravitational force, and the magnetic field, respectively; $\boldsymbol{u} = (u,v,w)$ and $\boldsymbol{x} = (x,y,z)$. Note that the signs $\mp$ correspond to the Walters B' and Rivlin-Ericksen models, respectively. The continuity equation of the medium is

$$\frac{\partial \rho}{\partial t} + (\boldsymbol{u}\cdot\nabla)\rho = 0. \tag{2}$$

For an incompressible medium we have

$$\nabla\cdot\boldsymbol{u} = 0. \tag{3}$$

The distance between the particles is assumed to be so large that interparticle contacts can be ignored. Buoyance and electrical forces on the particles are neglected. The density changes are small except in the gravity term. This is the Boussinesq approximation. Under these approximations, the equations of the motion and continuity for the particles are [42]

$$mN\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = KN(\boldsymbol{v}-\boldsymbol{u}), \tag{4}$$

$$\frac{\partial N}{\partial t} + \nabla\cdot(N\boldsymbol{v}) = 0, \tag{5}$$

where m is the mass of the particle.

Finally, Maxwell's equations for a conducting medium with finite resistivity η are [43]

$$\nabla\cdot\boldsymbol{H} = 0, \tag{6}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{H}) + \eta\nabla^2\boldsymbol{H}, \tag{7}$$

where $\eta = c^2/4\pi\sigma$. σ is the conductivity and c the velocity of light. Now let the initial state of the system be denoted by the subscript "0", and be a quiescent layer with uniform particle distribution. Thus, we have

$$\boldsymbol{u}_0 = 0,\ \boldsymbol{v}_0 = 0,\ N_0 = 0,\ \text{and}\ \boldsymbol{H}_0 = (H(z),0,0). \tag{8}$$

The physical origin of the field has to be created by currents outside the considered layer of thickness d. In this case, the magnetic field can be expressed by an exponential function of z, as well as a linear or parabolic function.

The stability of the initial state is studied by writing solutions to the full equations as the initial state plus a perturbation term (denoted by primes):

$$\boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{u}',\ \boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{v}',\ N = N_0 + N',\quad \boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{h}',\ p = p_0 + \delta p,\ \text{and}\ \rho = \rho_0 + \delta\rho, \tag{9}$$

where δp and $\delta\rho$ denote perturbations in pressure and density, respectively. Next, we put the perturbations assumed in (9) in the equations (1)–(7) and linearize them by neglecting products of the perturbations. We also drop the subscript "0" from the equilibrium quantities and the primes from the perturbed quantities, and obtain

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} = -\nabla\delta p + \boldsymbol{g}\delta\rho + \rho\left(\nu \mp \nu'\frac{\partial}{\partial t}\right)\nabla^2\boldsymbol{u} + KN(\boldsymbol{v}-\boldsymbol{u}) + \frac{\mu_e}{4\pi}[(\nabla\times\boldsymbol{h})\times\boldsymbol{H}_0 + (\nabla\times\boldsymbol{H}_0)\times h] + \left(\frac{\mathrm{d}\mu}{\mathrm{d}z} \mp \frac{\partial}{\partial t}\frac{\mathrm{d}\mu'}{\mathrm{d}z}\right)\left(\frac{\partial w}{\partial \boldsymbol{x}} + \frac{\partial \boldsymbol{u}}{\partial z}\right), \tag{10}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\boldsymbol{v} = \boldsymbol{u}, \tag{11}$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = (\boldsymbol{H}_0\cdot\nabla)\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\boldsymbol{H}_0 + \eta\nabla^2\boldsymbol{h}, \tag{12}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{13}$$

$$\nabla\cdot\boldsymbol{h} = 0, \tag{14}$$

$$\frac{\partial\delta\rho}{\partial t} + (\boldsymbol{u}\cdot\nabla)\rho = 0, \tag{15}$$

where $\tau = m/K$ denotes the relaxation time for the suspended particles. We assume that the perturbations vary as

$$f(z)\exp(ik_x x + ik_y y + \sigma t), \tag{16}$$

where k_x and k_y are the wavenumbers along x and y directions, respectively, $k = \sqrt{k_x^2 + k_y^2}$, σ is the growth rate of the perturbation, and $f(z)$ is some function of z.

Substituting $\boldsymbol{v}$ from equation (11) into (10), and then employing (15) and (16), we obtain the components of (10) as follows:

$$[\rho(1+\sigma\tau)+mN]\sigma u = -(1+\sigma\tau)(ik_x\delta p) + \rho(\nu \mp \nu'\sigma)(1+\sigma\tau)(D^2-k^2)u + (1+\sigma\tau)(ik_x w + Du)(D\mu \mp \sigma D\mu') + \frac{\mu_e}{4\pi}(1+\sigma\tau)(DH)h_z, \tag{17}$$

$$[\rho(1+\sigma\tau)+mN]\sigma v = -(1+\sigma\tau)(ik_y\delta p) + \rho(\nu \mp \nu'\sigma)(1+\sigma\tau)(D^2-k^2)v + (1+\sigma\tau)(ik_y w + Dv)(D\mu \mp \sigma D\mu') + \frac{\mu_e H}{4\pi}(1+\sigma\tau)(ik_x h_y - ik_y h_x), \tag{18}$$

$$[\rho(1+\sigma\tau)+mN]\sigma y = -(1+\sigma\tau)(D\delta p) + \rho(\nu \mp \nu'\sigma)(1+\sigma\tau)(D^2-k^2)w + 2(1+\sigma\tau)(Dw)(D\mu \mp \sigma D\mu') + \frac{g}{\sigma}(1+\sigma\tau)(D\rho)w + \frac{\mu_e H}{4\pi}(1+\sigma\tau)(ik_x h_z - Dh_x) - \frac{\mu_e}{4\pi}(1+\sigma\tau)(DH)h_x, \tag{19}$$

where $D = \mathrm{d}/\mathrm{d}z$. Similarly, using (16), one obtains from equations (12)–(15)

$$(\sigma - \eta\nabla^2)h_x = ik_x Hu - wDH, \tag{20}$$

$$(n - \eta\nabla^2)h_y = ik_x Hv, \tag{21}$$

$$(\sigma - \eta\nabla^2)h_z = ik_x Hw, \tag{22}$$

$$\sigma\delta\rho = -wD\rho, \tag{23}$$

$$ik_x u + ik_y v + Dw = 0, \tag{24}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \tag{25}$$

where $\nabla^2 = (D^2 - k^2)$.

3. The Dispersion Relations

It is easy to eliminate δp from (19) with the help of (17) and (18). Substitution of h_z from (22) in the resulting equation yields the following differential equation in the perturbed velocity component w:

$$\begin{aligned}&\sigma[(1+\sigma\tau)+\alpha_0][D(\rho Dw)-k^2\rho w]\\ &-(1+\sigma\tau)(\mu \mp \mu'\sigma)(D^2-k^2)^2 w\\ &-(1+\sigma\tau)(D^2-k^2)(D\mu \mp \sigma D\mu')Dw\\ &-(1+\sigma\tau)(D^2\mu \mp \sigma D^2\mu')(D^2+k^2)w\\ &+\frac{\mu_e k_x^2(1+\sigma\tau)}{4\pi(\sigma-\eta\nabla^2)}[H^2(D^2-k^2)w+(DH^2)(Dw)]\\ &+\frac{gk^2}{\sigma}(1+\sigma\tau)(D\rho)w = 0,\end{aligned} \tag{26}$$

where $\alpha_0 = mN/\rho$ denotes the mass concentration of the suspended particles. Now we shall solve (26) for a gas-particle medium which is a continuous stratification of the density $\rho(z)$ given in [44]

$$\rho(z) = \begin{cases} \rho_0\exp(\beta z) & 0 \le z \le d, \\ 0 & \text{otherwise}, \end{cases} \tag{27}$$

where ρ_0 is the density of the lower boundary, β is a constant, and d is the depth of the medium. The assumed Boussinesq approximation is always compatible with (27), giving an exponential dependence for the density. We must note here that a similar stratification is assumed for to the suspended particles. For the sake of simplicity, we consider a similar stratification for the viscosity, viscoelasticity, and magnetic field, i. e.

$$\left[\mu(z), \mu'(z), H^2(z)\right] = \left[\mu_0, \mu_0', \widetilde{H}^2\right]\exp(\beta z). \tag{28}$$

This assumption is mathematically appealing, as one gets constant kinematic viscosities. It follows from (28) that the kinematic viscosity ν, the kinematic viscoelasticity ν', and the Alfvén velocity $V_\mathrm{A} = (\mu_e\widetilde{H}^2/4\pi\rho_0)^{1/2}$ are constants throughout the medium. Following Chandrasekhar [1], and Bhatia [45], the requisite boundary conditions, appropriate for a medium with both boundary surfaces free, are

$$w = 0,\ D^2 w = 0,\ D^4 w = 0 \text{ at } z = 0, \text{ and } z = d. \tag{29}$$

In fact w and its even derivatives should vanish at the boundaries in this case.

In the following analysis we have treated the problem under consideration of two free surfaces. It has pointed out by Spiegel [46] that free boundaries bear relevance for certain stellar atmospheres. Many authors, including Chandrasekhar [1], have adopted stress-free boundaries because they allow for exact solutions of the problem. We shall now obtain the dispersion relation from (26), which represents the combined influence of finite conductivity, viscosity, viscoelasticity, suspended particles, and the magnetic field. For this, we first employ (27) and (28) in (26), and then solve the resulting equation, neglecting the effect of heterogeneity on the inertia term. We thus obtain

$$\begin{aligned}&(\sigma\nabla^4-\eta\nabla^6)w\\&-\frac{\sigma[(1+\sigma\tau)+\alpha_0]}{(\nu_0\mp\nu_0'\sigma)(1+\sigma\tau)}(\sigma\nabla^2-\eta\nabla^4)w\\&-\frac{k_x^2V_\mathrm{A}^2}{(\nu_0\mp\nu_0'\sigma)}\nabla^2w-\frac{gk^2\beta}{(\nu_0\mp\nu_0'\sigma)}(\sigma-\eta\nabla^2)w=0,\end{aligned}\tag{30}$$

where $\nu_0=\mu_0/\rho_0$ and $\nu_0'=\mu_0'/\rho_0$.

The differential equation (30), which contains only even derivatives of w, has to be solved consistent with the boundary condition stated in (29), i. e.

$$w=0,\ D^2w=0 \text{ at } z=0, \text{ and } z=d. \tag{31}$$

We find that w and all its even derivatives vanish at $z=0$ and $z=d$. The proper solution of (30) in view of the above boundary condition is

$$w=A\sin(\widetilde{m}\pi z/d). \tag{32}$$

From (30), inserting the values of w and its derivatives in accordance to (32), we obtain the dispersion relation

$$\begin{aligned}&\sigma(1+\sigma\tau)(\nu_0\mp\nu_0'\sigma)(\sigma L^2+\eta L^3)\\&+\sigma^2(1+\sigma\tau+\alpha_0)(\sigma L+\eta L^2)\\&-gk^2\beta(1+\sigma\tau)(\sigma+\eta L)+k_x^2V_\mathrm{A}^2\sigma L(1+\sigma\tau)=0,\end{aligned}\tag{33}$$

where $L=(\widetilde{m}\pi/d)^2+k^2$, and $\widetilde{m}$ is a constant.

It can be easily verified that in the absence of kinematic viscoelasticity, (33) reduces to the dispersion relation obtained by Sanghvi and Chhajlani [39], and that for an ideal plasma ($\eta=0$) in which $\nu_0'=0$, it reduces to the dispersion relation obtained by Chhajlani et al. [25]. Equation (33) can be further simplified and cast in the form

$$\begin{aligned}&\tau\sigma^4[1\mp\nu_0'L]\\&+\sigma^3[\tau L(\nu_0+\eta)+(1+\alpha_0)\mp\nu_0'L(1+\eta\tau L)]\\&+\sigma^2[L\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0\tau\mp\nu_0')+\tau\lambda]\\&+\sigma[\eta(\nu_0L^2-gk^2\beta\tau)+\lambda]-gk^2\beta\eta=0,\end{aligned}\tag{34}$$

where $\lambda=k_x^2V_\mathrm{A}^2-(gk^2\beta/L)$. Here we introduce the relaxation frequency parameter $f=1/\tau$ of the suspended particles. Then (34) assumes the form

$$\begin{aligned}&\sigma^4[1\mp\nu_0'L]\\&+\sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)\mp\nu_0'L(f+\eta L)]\\&+\sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0\mp\nu_0'f)+\lambda]\\&+\sigma[\eta(\nu_0fL^2-gk^2\beta)+\lambda f]-gk^2\beta\eta f=0,\end{aligned}\tag{35}$$

which is the general dispersion relations (where the signs $\mp$ correspond to the dispersion relations for the Walters B' and Rivlin-Ericksen models, respectively) representing the combined influence of viscosity, viscoelasticity, suspended particles, finite resistivity, and stratification parameter on the Rayleigh-Taylor instability of a composite gas-particle magnetized medium for the two cases.

4. Walters B' Viscoelastic Model

For the case of a stratified Walters B′ viscoelastic dusty plasma with finite resistivity, (35) can be written in the form

$$\begin{aligned}&\sigma^4[1-\nu_0'L]\\&+\sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)-\nu_0'L(f+\eta L)]\\&+\sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0-\nu_0'f)+\lambda]\\&+\sigma[\eta(\nu_0fL^2-gk^2\beta)+\lambda f]-gk^2\beta\eta f=0.\end{aligned}\tag{36}$$

In the following we shall discuss analytically and numerically the stability analysis for the Walters B' viscoelastic model, and obtain the stability conditions for such a model. We shall also discuss the corresponding case of an infinitely conducting dusty plasma.

4.1. Stability Conditions

It is elucidating to discuss (36) for the following two special cases:

(i) Longitudinal mode ($k_x = k$, $k_y = 0$). This case concerns perturbations parallel to the magnetic field. The dispersion relation (36) in this case becomes

$$\begin{aligned}&\sigma^4[1-\nu_0'L]\\&+\sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)-\nu_0'L(f+\eta L)]\\&+\sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0-\nu_0'f)+\widetilde{\lambda}]\\&+\sigma[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda}f]-gk^2\beta\eta f=0,\end{aligned} \tag{37}$$

where $\widetilde{\lambda} = k^2V_A^2 - (gk^2\beta/L)$.

Now, when $\beta < 0$ (stable stratification), all coefficients of (37) are real and positive if the following conditions are satisfied simultaneously:

$$\nu_0'L < 1, \tag{38}$$

$$\nu_0'L(f+\eta L) < [L(\nu_0+\eta)+f(1+\alpha_0)], \tag{39}$$

$$\left[\nu_0'f\eta L^2+\frac{gk^2\beta}{L}\right] < [fL\{\nu_0+\eta(1+\alpha_0)\} + \eta L^2\nu_0 + k^2V_A^2], \tag{40}$$

$$\frac{gk^2\beta}{L}(f+\eta L) < f[k^2V_A^2+\nu_0\eta L^2], \tag{41}$$

and hence (36) will not admit any real positive root, or complex root with a positive real part, implying thereby stability of the considered system. Otherwise, the system will be unstable. When $\beta > 0$, which is the criterion for unstable density stratification, the constant term in (37) is negative. Hence (37) has at least one positive real root leading to instability of the system.

(ii) Transverse mode ($k_x = 0$, $k_y = k$). Here we consider perturbations which are perpendicular to the direction of the magnetic field. The dispersion relation (36) for this particular mode can be written in the form

$$\begin{aligned}&\sigma^4[1-\nu_0'L]\\&+\sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)-\nu_0'L(f+\eta L)]\\&+\sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0-\nu_0'f)\\&\qquad-(gk^2\beta/L)]\\&+\sigma[\eta(\nu_0 fL^2-gk^2\beta)-(gk^2\beta f/L)]\\&-gk^2\beta\eta f=0.\end{aligned} \tag{42}$$

When $\beta < 0$ (stable stratification), all the coefficients of (42) are real and positive, and hence the system is stable, if the following conditions are simultaneously satisfied:

$$\nu_0'L < 1 \tag{43}$$

$$\nu_0'L(f+\eta L) < [L(\nu_0+\eta)+f(1+\alpha_0)], \tag{44}$$

$$\left[\nu_0'f\eta L^2+\frac{gk^2\beta}{L}\right] < [fL\{\nu_0+\eta(1+\alpha_0)\} + \eta L^2\nu_0], \tag{45}$$

$$\frac{gk^2\beta}{L}(f+\eta L) < f\nu_0\eta L^2. \tag{46}$$

When $\beta > 0$ (unstable stratification), the constant term in (42) is negative. Hence (42) has at least one positive real root, leading thereby to instability of the system.

Henceafter we will deal with the general dispersion relation (36), since the case of transverse perturbations is relatively unimportant, which is evident from the foregoing discussion, because the magnetic field does not affect the stability of the system in this case. Now we shall estimate the growth rate of the unstable Rayleigh-Taylor modes. It is clear from (37) that in this case ($\beta > 0$) the general dispersion relation (37) will possess at least one positive real root which leads to instability. Let us denote this root by σ_0, then σ_0 will satisfy the equation

$$\begin{aligned}&\sigma_0^4[1-\nu_0'L]\\&+\sigma_0^3[L(\nu_0+\eta)+f(1+\alpha_0)-\nu_0'L(f+\eta L)]\\&+\sigma_0^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0-\nu_0'f)+\widetilde{\lambda}]\\&+\sigma_0[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda}f]-gk^2\beta\eta f=0.\end{aligned} \tag{47}$$

Now, to comprehend the implifications of finite resistivity, suspended particles, viscosity, viscoelasticity of the medium and the stratification parameter on the growth rate of the unstable Rayleigh-Taylor modes, we evaluate $\mathrm{d}\sigma_0/\mathrm{d}f$, $\mathrm{d}\sigma_0/\mathrm{d}\nu_0$, $\mathrm{d}\sigma_0/\mathrm{d}\nu_0'$, $\mathrm{d}\sigma_0/\mathrm{d}V_A^2$, and $\mathrm{d}\sigma_0/\mathrm{d}\beta$ from (47), and discuss their nature. Assuming that both k and α_0 are constants, we get

$$\begin{aligned}\frac{\mathrm{d}\sigma_0}{\mathrm{d}f} = -\frac{1}{F}[&\sigma_0^3\{(1+\alpha_0)-\nu_0'L\}\\&+\sigma_0^2L\{\nu_0+\eta(1+\alpha_0)-\nu_0'\eta L\}\\&+\sigma_0(\nu_0\eta L^2+\widetilde{\lambda})-gk^2\beta\eta],\end{aligned} \tag{48}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0} = -\frac{\sigma_0 L}{F}(\sigma_0+f)(\sigma_0+\eta L), \tag{49}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0'} = \frac{\sigma_0^2 L}{F}(\sigma_0+f)(\sigma_0+\eta L), \tag{50}$$

$$\frac{d\sigma_0}{d\eta} = -\frac{L}{F}[\sigma_0^3(1-v_0'L) + \sigma_0^2\{f(1+\alpha_0)+L(v_0-v_0'f)\} + \sigma_0\{v_0 fL-(gk^2\beta/L)\} - f(gk^2\beta/L)], \quad (51)$$

$$\frac{d\sigma_0}{dV_A^2} = -\frac{\sigma_0 k^2}{F}(\sigma_0+f), \quad (52)$$

$$\frac{d\sigma_0}{d\beta} = \frac{gk^2}{FL}[\sigma_0^2+f\sigma_0+f\eta L], \quad (53)$$

where

$$F = 4\sigma_0^3[1-v_0'L] + 3\sigma_0^2[L(v_0+\eta)+f(1+\alpha_0)-v_0'L(f+\eta L)] + 2\sigma_0[fL\{v_0+\eta(1+\alpha_0)\}+\eta L^2(v_0-v_0'f)+\widetilde{\lambda}] + [\eta(v_0 fL^2-gk^2\beta)+\widetilde{\lambda}f]. \quad (54)$$

Now, consider the inequalities

$$(\sigma_0+\eta L)[\sigma_0^2 v_0'L+(gk^2\beta/L)] \lessgtr [\sigma_0^3\{(1+\alpha_0)+\sigma_0^2 L\{v_0+\eta(1+\alpha_0)\} + \sigma_0\{v_0\eta L^2+k^2V_A^2], \quad (55)$$

$$[4\sigma_0^3 v_0'L+3\sigma_0^2 v_0'L(f+\eta L) + 2\sigma_0\{\eta L^2 v_0' f+(gk^2\beta/L)\}] \lessgtr [4\sigma_0^3+3\sigma_0^2\{L(v_0+\eta)+f(1+\alpha_0)\} + 2\sigma_0\{fL(v_0+\eta(1+\alpha_0))+\eta L^2 v_0+k^2V_A^2\} + \{\eta v_0 fL^2+fk^2V_A^2\}], \quad (56)$$

$$[\sigma_0^3 v_0'L+\sigma_0^2 v_0' fL+(gk^2\beta/L)(\sigma_0+f)] \lessgtr \sigma_0[\sigma_0^2+\sigma_0\{f(1+\alpha_0)+v_0 L\}+v_0 fL]. \quad (57)$$

If either both the upper or both the lower signs of the inequalities (55) and (56) simultaneously hold, then $d\sigma_0/df$, given by (48) is negative. Thus we infer that the growth rate of the unstable Rayleigh-Taylor modes decreases with increasing the relaxation frequency parameter of the suspended particles, when the mentioned restrictions hold. Thus the conditions (55) and (56) define the region where the suspended particles have a stabilizing influence. But if the upper sign of the inequality (55), and the lower sign of the inequality (56), or vice-versa, hold simultaneously, then the growth rate turns out to be positive. This means, under certain limitations, that the suspended particles can increase the growth rate of the unstable Rayleigh-Taylor modes. We observe from (55) and (56) that the stabilizing or destabilizing influence of the suspended particles is dependent on the finite resistivity of the medium, the kinematic viscosity, the kinematic viscoelasticity, and the magnetic field. Also, if either both the upper or both the lower signs of the inequalities (56) and (57) simultaneously hold, then $d\sigma_0/d\eta$, given by (51) is negative. Thus we infer that the growth rate of the unstable Rayleigh-Taylor modes decreases with increasing the finite resistivity, when the mentioned restrictions hold. Thus the conditions (56) and (57) define the region where the finite resistivity has a stabilizing influence. But if the upper sign of the inequality (56), and the lower sign of the inequality (57), or vice-versa, hold simultaneously, then the growth rate turns out to be positive. This means, under certain limitations, that the finite resistivity can increase the growth rate of the unstable Rayleigh-Taylor modes. We observe from (56) and (57) that the stabilizing or destabilizing influence of the finite resistivity depends on the relaxation frequency parameter of the suspended particles, the kinematic viscosity, and the kinematic viscoelasticity as well as the applied magnetic field.

From (49), (52), and (56) it is clear that both $d\sigma_0/dv_0$ and $d\sigma_0/dV_A^2$ are negative or positive if $F \gtrless 0$, respectively, i.e. if the upper or lower sign of the inequality (56) holds, respectively. This means that the growth rate of the unstable mode is negative if $F > 0$, and it is positive if $F < 0$. Both the kinematic viscosity and the magnetic field have stabilizing effects in the former case, while they have destabilizing influences in the later case. Also, from (50), (53), and (56) it is clear that both $d\sigma_0/dv_0'$ and $d\sigma_0/d\beta$ are negative or positive according to $F \lessgtr 0$, respectively, i.e. if the lower or upper sign of the inequality (56) holds. This means that the growth rate of the unstable mode is positive if $F > 0$, and it is negative if $F < 0$. Both the kinematic viscoelasticity and the stratification parameter destabilize in the former case, while they stabilizing in the latter case. Thus we conclude that all the physical parameters included in this work have stabilizing as well as destabilizing effects, depending on the other quantities under certain conditions, as shown in the previous limitations (55) – (57).

4.2. Numerical Discussion

Equation (36) ia a quadruple equation in the growth rate σ, with real coefficients. We solved this equation numerically (using Mathematica 4) for $\widetilde{m} = 1$, $d = 2$ m, and $g = 980$ m s^{-2}, for various values of the physical parameters α_0, η, V_A, ν_0, f, ν_0', and β, in the case of potentially unstable stratification ($\beta > 0$). These calculations are presented in Figs. 1 – 7, where we have shown the growth rate (positive real part of σ) against the wavenumber k (from $k = 0$ to $k \le 10$) for α_0 (the mass concentration of the suspended particles) = 0.7, 3, and 10 kg m^{-2}; ν_0 (the kinematic viscosity) = 0.5, 3, and 10 m^2 s^{-1}; η (the finite resistivity) = 0.4, 2, and 5 kg m^3 A^{-2} s^{-3}; f (the relaxation frequency of the suspended particles) = 5, 10, and 15 kg^{-1} m^{-1}; V_A (the Alfvén velocity) = 10, 40, and 100 m s^{-1}; β (the stratification parameter) = 0.1, 0.8, and 1.5; and ν_0' (the kinematic viscoelasticity) = 0.7, 1.2, and 2 m^2 s^{-1}, respectively.

It can be seen from Figs. 1 and 2 that, as both the mass concentration of the suspended particles α_0, and the kinematic viscosity ν_0 are increasing, the growth rate σ is increased, showing thereby the destabilizing character of the effects of both α_0 and ν_0, respectively. Figs. 1 and 2 show also, for small fixed values of α_0 and ν_0, respectively, that the growth rate increases for small wavenumbers up to a critical wavenumber, after which the growth rate decreases for all the next wavenumbers. There are also critical values of both α_0 and ν_0, after which the growth rate σ decreases for all wavenumbers. Therefore, for fixed small values of α_0 or ν_0, the system is found to have stabilizing as well as a destabilizing, effects depending on the range of the wavenubers, in the presence of a finite resistivity and stratification parameter. Note that for wavenumbers $k \ge 10$ in Fig. 1, all the curves coincide for all values of α_0, and this indicates that the mass concentration of the suspended particles has no effect on the stability of the system in this case, and the system is stable irrespective of the α_0 values.

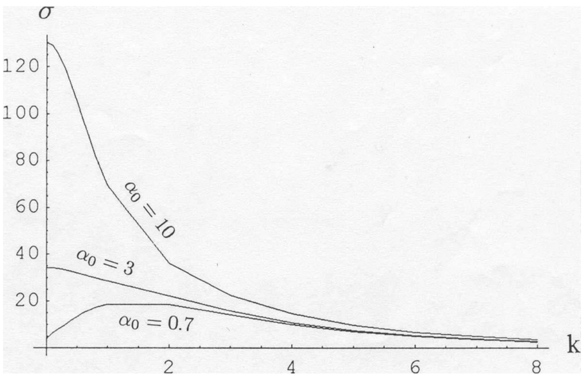

Fig. 1. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the mass concentration of the suspended particles $\alpha_0 =$ 0.7, 3, and 10 kg m^{-2}, with $\widetilde{m} = 1$, $d = 2$ m, $\nu_0 = 0.7$ m^2 s^{-1}, $\nu_0' = 0.7$ m^2 s^{-1}, $\eta = 6$ kg m^3 A^{-2} s^{-3}, $f = 10$ kg^{-1} m^{-1}, $\beta = 0.2$, $V_A = 30$ m s^{-1}, and $g = 980$ m s^{-2}.

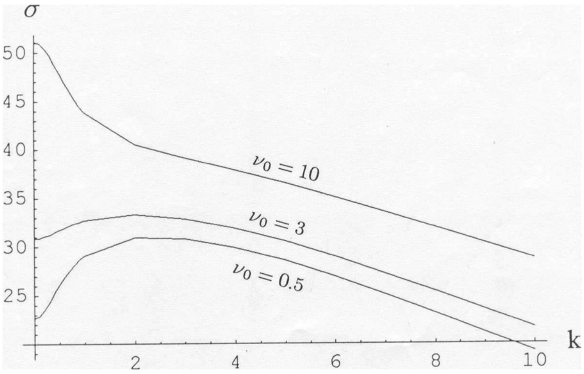

Fig. 2. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the kinematic viscosity $\nu_0 = 0.5$, 3, and 10 m^2 s^{-1}, with $\widetilde{m} = 1$, $d = 2$ m, $\alpha_0 = 3$ kg m^{-2}, $\nu_0' = 0.5$ m^2 s^{-1}, $\eta = 0.4$ kg m^3 A^{-2} s^{-3}, $f = 5$ kg^{-1} m^{-1}, $\beta = 0.2$, $V_A =$ 20 m s^{-1}, and $g = 980$ m s^{-2}.

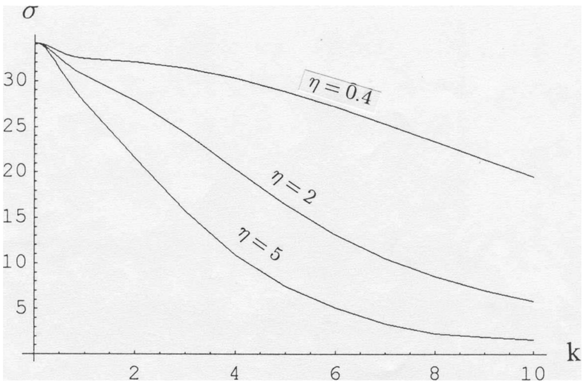

Fig. 3. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the finite resistivity $\eta = 0.4$, 2, and 5 kg m^3 A^{-2} s^{-3}, with $\widetilde{m} = 1$, $d = 2$ m, $\nu_0 = 0.8$ m^2 s^{-1}, $\nu_0' = 0.8$ m^2 s^{-1}, $\alpha_0 = 3$ kg m^{-2}, $f = 15$ kg^{-1} m^{-1}, $\beta = 0.3$, $V_A = 30$ m s^{-1}, and $g = 980$ m s^{-2}.

Figs. 3 and 4 show that, as both the finite resistivity η and the relaxation frequency of the suspended particles f are increasing, the growth rate σ decreases

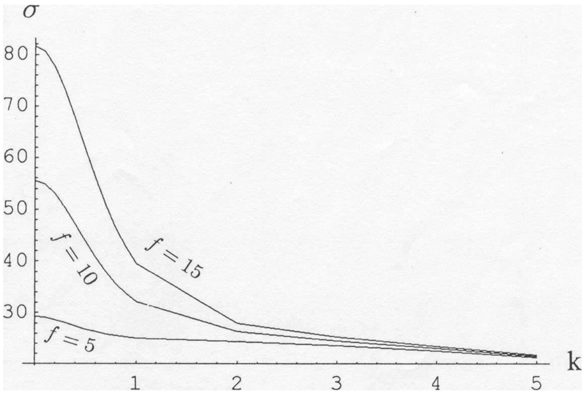

Fig. 4. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the relaxation frequency of the suspended particles $f = 5$, 10, and 15 $\text{kg}^{-1}\,\text{m}^{-1}$, with $\widetilde{m} = 1$, $d = 2$ m, $v_0 = 0.5\ \text{m}^2\,\text{s}^{-1}$, $v_0' = 0.8\ \text{m}^2\,\text{s}^{-1}$, $\eta = 0.4\ \text{kg}\,\text{m}^3\,\text{A}^{-2}\,\text{s}^{-3}$, $\alpha_0 = 3\ \text{kg}\,\text{m}^{-2}$, $\beta = 0.2$, $V_\text{A} = 30\ \text{m}\,\text{s}^{-1}$, and $g = 980\ \text{m}\,\text{s}^{-2}$.

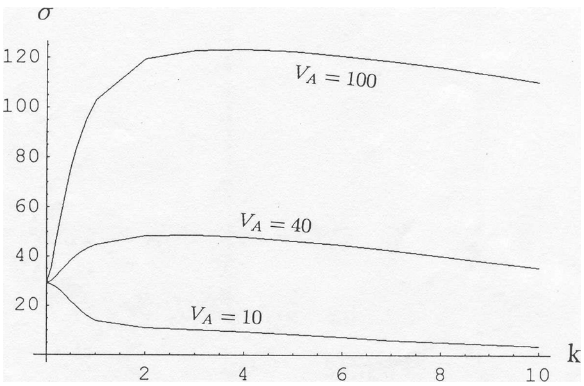

Fig. 5. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the Alfvén velocity $V_\text{A} = 10$, 40, and 100 $\text{m}\,\text{s}^{-1}$, with $\widetilde{m} = 1$, $d = 2$ m, $v_0 = 0.5$, $v_0' = 0.6$, $\eta = 0.4\ \text{kg}\,\text{m}^3\,\text{A}^{-2}\,\text{s}^{-3}$, $f = 5\ \text{kg}^{-1}\,\text{m}^{-1}$, $\beta = 0.2$, $\alpha_0 = 3\ \text{kg}\,\text{m}^{-2}$, and $g = 980\ \text{m}\,\text{s}^{-2}$.

and increases, respectively, showing thereby the stabilizing effect of η, and the destabilizing effect of f. We note from Fig. 3, that the growth rate is constant ($\sigma \approx 34$) for zero wavenumber, since all the curves in the figure start from the same point; and for a constant value of finite resistivity η, the considered system is found to have a stabilizing effect in this case in the presence of the other physical quantities. We note also from Fig. 4, that for a constant relaxation frequency of the suspended particles, the considered system has a stabilizing influence. It follows also that the relaxation frequency of the suspended particles has no effect on the stability of the system for the wavenymbers $k \geq 5$, since all curves coincide at starting point of this minimum wavenumber, and the system in this case is stable irrespective of the f values.

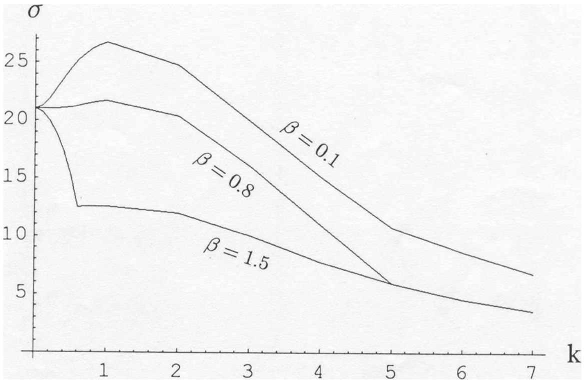

Fig. 6. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the stratification parametrt $\beta = 0.1$, 0.8, and 1.5, with $\widetilde{m} = 1$, $d = 2$ m, $v_0 = 0.7\ \text{m}^2\,\text{s}^{-1}$, $v_0' = 0.7\ \text{m}^2\,\text{s}^{-1}$, $\eta = 4\ \text{kg}\,\text{m}^3\,\text{A}^{-2}\,\text{s}^{-3}$, $f = 10\ \text{kg}^{-1}\,\text{m}^{-1}$, $\alpha_0 = 2\ \text{kg}\,\text{m}^{-2}$, $V_\text{A} = 30\ \text{m}\,\text{s}^{-1}$, and $g = 980\ \text{m}\,\text{s}^{-2}$.

It can be seen from Figs. 5 and 6 that as both the Alfvén velocity V_A, and the stratification parameter β are increasing, the growth rate σ increases and decreases, respectively, showing thereby the destabilizing character of the effect of the Alfvén velocity V_A and the stabilizing character of the effect of the stratification parameter β in the presence of fluid resistivity and kinematic viscoelasticity. It should be noted, from Fig. 5, that the system is stabilized for small constant Alfvén velocities, where the growth rate decreases for all wavenumbers; while for large constant Alfvén velocities it is found that the system is unstable (for small wavenumbers) as well as stable (for large wavenumbers), where the growth rate increases with increasing the wavenumbers till a critical wavenumber, after which the growth rate decreases with increasing wavenumber k. We note also, from Fig. 6, that for small values of the stratification parameter β, the system is unstable (for small wavenumbers) and stable (for large wavenumbers), where the growth rate increases with increasing wavenumbers till a critical wavenumber, after which the growth rate decreases for all wavenumbers greater than the critical one. For larger stratification parameters β, i.e. for $\beta \geq 1.5$, it is found that the system is always stable in the presence of all considered physical parameters.

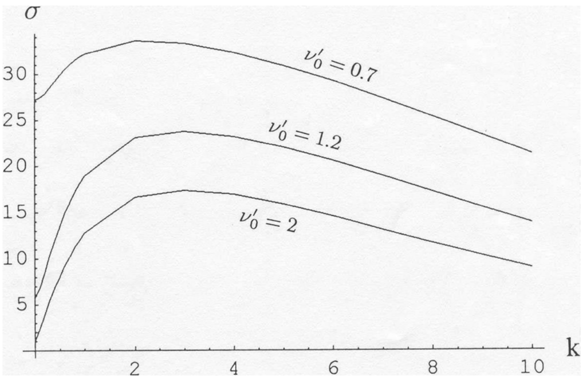

Fig. 7. The positive real part of the growth rate σ, given by (36), of the unstable mode plotted against the wavenumber k for the kinematic viscoelasticity $\nu_0' = 0.7$, 1.2, and 2 $\mathrm{m^2\,s^{-1}}$, with $\widetilde{m} = 1$, $d = 2$ m, $\nu_0 = 0.5$ $\mathrm{m^2\,s^{-1}}$, $\eta = 0.4$ $\mathrm{kg\,m^3\,A^{-2}\,s^{-3}}$, $\alpha_0 = 3$ $\mathrm{kg\,m^{-2}}$, $f = 8$ $\mathrm{kg^{-1}\,m^{-1}}$, $\beta = 0.2$, $V_A = 30$ $\mathrm{m\,s^{-1}}$, and $g = 980$ $\mathrm{m\,s^{-2}}$.

Finally, Fig. 7 shows that as the kinematic viscoelasticity ν_0' increases, the growth rate decreases, showing thereby the stabilizing the effect of the kinematic viscoelasticity ν_0', while for a fixed kinematic viscoelasticity it is found that the system is destabilizing (for small wavenumbers) as well as stabilizing (for large wavenumbers) in the presence of suspended particles, magnetic field, and density stratification parameters.

4.3. Dusty Plasma with Infinite Conductivity

The present section concerns the absence of a magnetic field and may be particularly useful for astrophysical plasmas where the approximation of infinite conductivity is considered valid, On substituting $\eta = 0$ and $V_A = 0$, we find that the dispersion relation (36) reduces to

$$\sigma^3[1-\nu_0'L] + \sigma^2[\nu_0 L + f(1+\alpha_0) - \nu_0' fL] + \sigma[\nu_0 fL - (gk^2\beta/L)] - f(gk^2\beta/L) = 0 \quad (58)$$

when $\beta < 0$, and (58) will not admit any real positive root, or complex root with positive real part, if the condition $\nu_0' < (1/L)$, is satisfied, and so the system remains stable in this case. Otherwise if $\nu_0' < (1/L)$, the system is unstable. For $\beta > 0$, (58) will possess at least one real positive root, which will destabilize the system for all wavenumbers. Let σ_0 denote the unstable real positive root of (58), then

$$\sigma_0^3[1-\nu_0'L] + \sigma_0^2[\nu_0 L + f(1+\alpha_0) - \nu_0' fL] + \sigma_0[\nu_0 fL - (gk^2\beta/L)] - f(gk^2\beta/L) = 0. \quad (59)$$

We also note from (59) that the unstable density stratification ($\beta > 0$) can not be stabilized. Now (59) yields

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}f} = -\frac{1}{G}[\sigma_0^2\{(1+\alpha_0) - \nu_0'L\} + \sigma_0(\nu_0 L) - (gk^2\beta/L)], \quad (60)$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0} = -\frac{\sigma_0 L}{G}(\sigma_0 + f), \quad (61)$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0'} = \frac{\sigma_0^2 L}{G}(\sigma_0 + f), \quad (62)$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\beta} = \frac{gk^2}{GL}(\sigma_0 + f), \quad (63)$$

where

$$G = 3\sigma_0^2[1-\nu_0'L] + 2\sigma_0[\nu_0 L + f(1+\alpha_0) - \nu_0' fL] + [\nu_0 fL - (gk^2\beta/L)]. \quad (64)$$

Note that $\mathrm{d}\sigma_0/\mathrm{d}\nu_0'$ and $\mathrm{d}\sigma_0/\mathrm{d}\beta$, given by (62) and (63), are positive or negative according to $G \gtreqless 0$, i.e. if the upper or lower sign of the inequality

$$[\sigma_0 \nu_0' L(3\sigma_0 + 2f) + (gk^2\beta/L)] \lesseqgtr [3\sigma_0^2 + 2\sigma_0\{\nu_0 L + f(1+\alpha_0)\} + \nu_0 fL] \quad (65)$$

holds, respectively. In this case, according to (65), $\mathrm{d}\sigma_0/\mathrm{d}\nu_0'$, given by (61) is negative or positive, respectively.

We thus infer that the kinematic viscosity ν_0, kinematic viscoelasticity ν_0', and the stratification parameter β can reduce as well as increase the growth rate of the unstable Rayleigh-Taylor perturbations as determined by (65). Note also that $\mathrm{d}\sigma_0/\mathrm{d}f$, given by (60), is always negative if the upper and lower signs of the inequalities (65) and

$$[\sigma_0^2 \nu_0 L + (gk^2\beta/L)] \lesseqgtr [\sigma_0^2(1+\alpha_0) + \sigma_0 \nu_0 L] \quad (66)$$

hold simultaneously. Thus the relaxation frequency of the suspended particles has a stabilizing effect in this case. Otherwise, it has a destabilizing influence on the considered system.

In the derivation of the above conditions (65) and (66) we have made use of the fact that $\alpha_0 = (mN/\rho)$, which is the mass concentration of the dust, can not exceed one. Thus we are led to conclude that the presence of dust always reduces the growth rate of the unstable Rayleigh-Taylor perturbations when the finite resistivity vanishes. A similar consequence of the presence of dust has been reported by Michael [47] in the context of Kelvin-Helmholtz instability.

5. Rivlin-Ericksen Viscoelastic Model

For the case of a stratified Rivlin-Ericksen viscoelastic dusty plasma with finite resistivity, (35) can be written in the form (considering the case of a longitudinal mode, i. e. when $k_x = k$)

$$\begin{aligned}&\sigma^4[1+\nu_0 L]\\&+\sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)+\nu_0' L(f+\eta L)]\\&+\sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0+\nu_0' f)+\lambda]\\&+\sigma[\eta(\nu_0 fL^2-gk^2\beta)+\lambda f]-gk^2\beta\eta f=0.\end{aligned} \tag{67}$$

As we know, the Rivlin-Ericksen fluid as viscoelastic fluids is modeled by the following constitutive equation [48]

$$\tau_{ij} = -p\delta_{ij} + \nu A_{ij}^{(1)} + \nu' A_{ij}^{(2)} + \nu'' A_{ik}^{(1)} A_{kj}^{(1)},$$

where

$$A_{ij}^{(1)} = u_{i,j} + u_{j,i},$$
$$A_{ij}^{(2)} = A_{ij,t}^{(1)} + u_m A_{ij,m}^{(1)} + A_{im}^{(1)} u_{m,j} + A_{mj}^{(1)} u_{m,i}.$$

Here τ_{ij} is the stress tensor, p the pressure, δ_{ij} the Kronecker delta, u_i the fluid velocity, and ν, ν', ν'' are three measurable material constants (viscosity, elasticity, and cross-viscosity).

Now, when $\beta < 0$ (stable stratification), all the coefficients of (67) are real and positive, and hence (67) will not admit any positive real root, or complex root with a real positive part, implying thereby stability of the considered system. We conclude that a stable density stratification will remain stable even for a finite resistivity together with suspended particles, and a nonzero kinematic viscosity and kinematic viscoelasticity. But when $\beta > 0$ which is the criterion for unstable density stratification, (67) will possess at least one real positive root, which will destabilize the considered system for all wavenumbers. Let σ_0 denote the unstable real positive root of (67), then it will satisfy the equation

$$\begin{aligned}&\sigma_0^4[1+\nu_0' L]\\&+\sigma_0^3[L(\nu_0+\eta)+f(1+\alpha_0)+\nu_0' L(f+\eta L)]\\&+\sigma_0^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\eta L^2(\nu_0+\nu_0' f)+\widetilde{\lambda}]\\&+\sigma_0[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda} f]-gk^2\beta\eta f=0.\end{aligned} \tag{68}$$

From (68), it follows that the unstable density stratification ($\beta > 0$) can not be stabilized, since the constant term of (68) is always negative in this case. Now to understand the influence of suspended particles with their relaxation frequency parameter f, of the kinematic viscosity ν_0, kinematic viscoelasticity of the medium ν_0', finite resistivity η, stratification parameter β, and the Alfvén velocity V_A, on the growth rate of the unstable Rayleigh-Taylor modes, we evaluate the following derivatives, using (68), and discuss their nature.

$$\begin{aligned}\frac{d\sigma_0}{df} = -\frac{1}{F_1}[&\sigma_0^3\{(1+\alpha_0)+\nu_0' L\}\\&+\sigma_0^2 L\{\nu_0+\eta(1+\alpha_0)+\nu_0'\eta L\}\\&+\sigma_0\{\nu_0\eta L^2+\widetilde{\lambda}\}-gk^2\beta\eta],\end{aligned} \tag{69}$$

$$\frac{d\sigma_0}{d\nu_0} = -\frac{\sigma_0 L}{F_1}[\sigma_0^2+\sigma_0(f+\eta L)+f\eta L], \tag{70}$$

$$\frac{d\sigma_0}{d\nu_0'} = \frac{\sigma_0^2 L}{F_1}[\sigma_0^2+\sigma_0(f+\eta L)+f\eta L], \tag{71}$$

$$\begin{aligned}\frac{d\sigma_0}{d\eta} = -\frac{L}{F_1}[&\sigma_0^3(1+\nu_0' L)\\&+\sigma_0^2\{f(1+\alpha_0)+L(\nu_0+\nu_0' f)\}\\&+\sigma_0\{\nu_0 fL-(gk^2\beta/L)\}-f(gk^2\beta/L)],\end{aligned} \tag{72}$$

$$\frac{d\sigma_0}{d\beta} = \frac{gk^2}{F_1 L}[\sigma_0^2+\sigma_0(f+\eta L)+f\eta L], \tag{73}$$

$$\frac{d\sigma_0}{dV_A} = -\frac{2\sigma_0 k_x^2 V_A}{F_1}(\sigma_0+f), \tag{74}$$

where

$$\begin{aligned}F_1 = {}&4\sigma_0^3[1+\nu_0' L]\\&+3\sigma_0^2[L(\nu_0+\eta)+f(1+\alpha_0)+\nu_0' L(f+\eta L)]\\&+2\sigma_0[fL\{\nu_0+\eta(1+\alpha_0)\}\\&\qquad+\eta L^2(\nu_0+\nu_0' f)+\widetilde{\lambda}]\\&+[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda} f].\end{aligned} \tag{75}$$

Now, consider the inequalities

$$\begin{aligned}&\frac{gk^2\beta}{L}(2\sigma_0+f+\eta L) \lesseqgtr\\&[4\sigma_0^3(1+\nu_0' L)\\&+3\sigma_0^2\{L(\nu_0+\eta)+f(1+\alpha_0)+\nu_0' L(f+\eta L)\}\\&+2\sigma_0\{fL(\nu_0+\eta(1+\alpha_0))+\eta L^2(\nu_0+\nu_0' f)\}\\&+\nu_0 f\eta L^2+k^2V_A^2],\end{aligned} \tag{76}$$

$$\frac{gk^2\beta}{L}(\sigma_0+\eta L) \lesseqgtr [\sigma_0^3\{(1+\alpha_0)+\nu_0' L\} + \sigma_0^2 L\{\nu_0+\eta(1+\alpha_0)+\nu_0'\eta L\} + \sigma_0\{\nu_0\eta L^2+k^2V_A^2\}], \tag{77}$$

and

$$\frac{gk^2\beta}{L}(\sigma_0+\eta L) \lesseqgtr [\sigma_0^3(1+\nu_0' L) + \sigma_0^2\{f(1+\alpha_0)+L(\nu_0+\nu_0' f)\}+\sigma_0\nu_0 fL], \tag{78}$$

resulting from the conditions

$$F_1 \gtrless 0, \tag{79}$$

$$[\sigma_0^3\{(1+\alpha_0)+\nu_0' L\} + \sigma_0^2 L\{\nu_0+\eta(1+\alpha_0)+\nu_0'\eta L\} + \sigma_0\{\nu_0\eta L^2+\widetilde{\lambda}\} - gk^2\beta\eta] \gtrless 0, \tag{80}$$

and

$$[\sigma_0^3(1+\nu_0' L) + \sigma_0^2\{f(1+\alpha_0)+L(\nu_0+\nu_0' f)\} + \sigma_0\{\nu_0 fL-(gk^2\beta/L)\} - f(gk^2\beta/L) \gtrless 0 \tag{81}$$

given by (69), (72), and (75), respectively.

5.1. Stability Analysis

If either both upper or both lower signs of the inequalities (76) and (77) hold, then $d\sigma_0/df$ is negative. Thus we infer that the growth rate of the unstable Rayleigh-Taylor modes decreases with increasing relaxation frequency parameter of the suspended particles when the above mentioned restrictions (76) and (77) hold. Thus the conditions (76) and (77) define the region where the suspended particles have a stabilizing influence. But if the upper sign of the inequality (76) and the lower sign of (77), or vice versa, hold simultaneously, then the growth rate turns out to be positive. This means, under these limitations, that the suspended particles can increase the growth rate of the unstable Rayleigh-Taylor modes. We observe from (76) and (77) that the stabilizing or destabilizing influence of the suspended particles depends on the finite resistivity of the medium, as well as the kinematic viscosity and viscoelasticity. Also, if either both upper or both lower signs of the inequalities (76) and (78) simultaneously hold, then $d\sigma_0/d\eta$ is negative. Thus we infer that the growth rate of the unstable Rayleigh-Taylor modes decreases with increasing finite resistivity of the medium, when the above mentioned restrictions (76) and (78) hold. Thus the conditions (76) and (78) define the region where the finite resistivity has a stabilizing influence. But if the upper sign of (76) and the lower sign of (78), or vice versa, hold simultaneously, then the growth rate turns out to be positive. This means, under these limitations, that the finite resistivity can increase the growth rate of the unstable Rayleigh-Taylor modes. Also, from (74) we find that the applied magnetic field has a stabilizing as well as a destabilizing effect on the considered system according as the upper and lower signs of (76), respectively.

From (70) and (75) we find that the growth rate of the unstable mode is negative or positive if $F \gtrless 0$, which correspond to the upper and lower signs of the inequality (76), respectively. It is also readily seen from (68) that for an infinitely conducting plasma ($\eta = 0$), the growth rate is always negative if the condition $V_A^2 > (g\beta/L)$ is satisfied, whereas for a finitely conducting medium we find from (70) that the growth rate is negative under the upper sign of (76). In the light of the above discussion it should, however, be observed that the kinematic viscosity always suppresses the growth rate of the unstable Rayleigh-Taylor mode for an ideal plasma, but for a finitely conducting plasma it enhances the growth rate under the upper sign of the condition (76). Note that the finite conductivity further destabilizes the system by increasing the growth rate of the unstable modes. Also, from (71), (73), and (75) we find that the growth rate of the unstable mode is positive or negative if $F \gtrless 0$, which corresponds to the upper and lower signs of the inequality (76), i.e. in the regions where the kinematic viscosity has a stabilizing effect, we find that both the kinematic viscoelasticity and stratification parameter have destabilizing influences on the considered system, and vice-versa.

Equation (67) ia a quadruple equation in the growth rate σ, with real coefficients. We solved this equation numerically (using Mathematica 4) for $\widetilde{m} = 1, d = 2$ m, and $g = 980$ m s^{-2}, for various values of the physical parameters α_0, η, V_A, ν_0, f, ν_0', and β, in the case of potentially unstable stratification ($\beta > 0$). These calculations are presented in Figs. 8 – 14, where we have given the growth rate (positive real part of σ) against the wavenumber k (from $k = 0$ to $k = 10$) for α_0 (the mass concentration of the suspended particles) $= 0.7$,

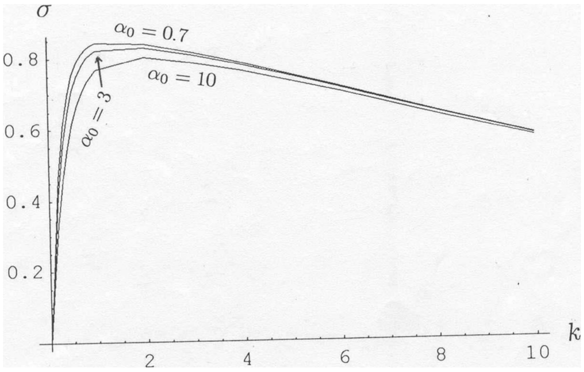

Fig. 8. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the mass concentration of the suspended particles $\alpha_0 = 0.7$, 3, and 10 kg m^{-2}, with the same values of the other parameters as given in Fig. 1.

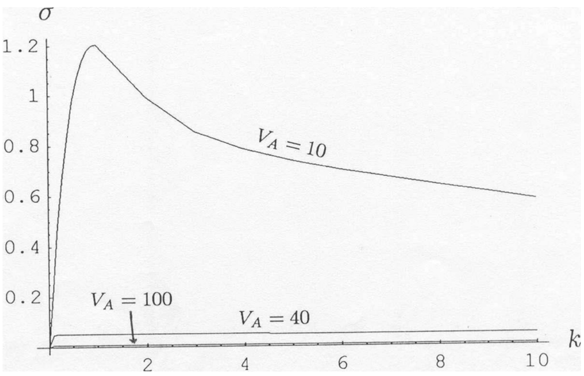

Fig. 9. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the the Alfvén velocity $V_A = 10$, 40, and 100 m s^{-1}, with the same values of the other parameters as given in Fig. 5.

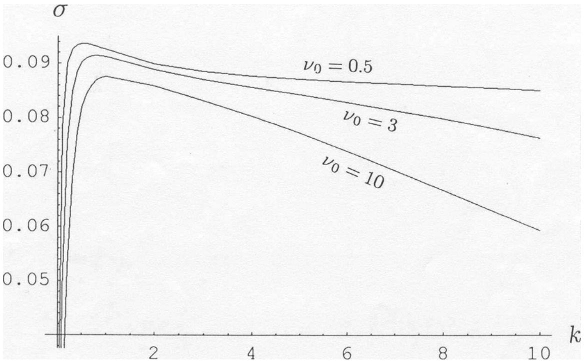

Fig. 10. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the kinematic viscosity $\nu_0 = 0.5$, 3, and 10 m^2 s^{-1}, with the same values of the other parameters as given in Fig. 2.

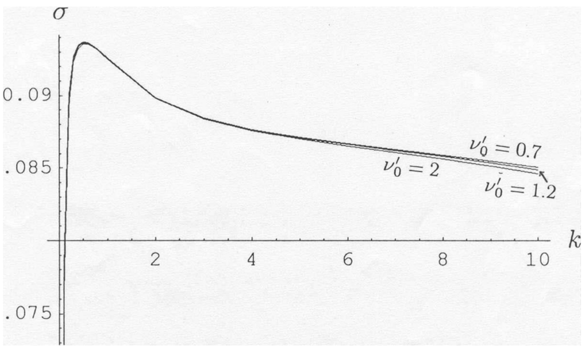

Fig. 11. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the kinematic viscoelasticity $\nu'_0 = 0.7$, 1.2, and 2 m^2 s^{-1}, with the same values of the other parameters as given in Fig. 7.

3, and 10 kg m^{-2}; V_A (the Alfvén velocity) $= 10$, 40, and 100 m s^{-1}; ν_0 (the kinematic viscosity) $= 0.5$, 3, and 10 m^2 s^{-1}; ν'_0 (the kinematic viscoelasticity) $= 0.7$, 1.2, and 2 m^2 s^{-1}; β (the stratification parameter) $=$ 0.1, 0.8, and 1.5; η (the finite resistivity) $= 0.4$, 2, and 5 kg m^3 A^{-2} s^{-3}; and f (the relaxation frequency of the suspended particles) $= 5$, 10, and 15 kg^{-1} m^{-1}; respectively. Figures 8 – 11 show that the considered Rivlin-Ericksen viscoelastic medium including the effects of magnetic field, finite resistivity, kinematic viscosity and viscoelasticity, and density stratification has destabilizing effects (for very small wavenumbers) as well as stabilizing effects (for large wavenumbers greater than critical wavenumbers) for fixed values of the parameters α_0, V_A, ν_0, ν'_0, β, η, and f, as indicated in the previous section. It follows from, Figs. 8 – 11, that the mass concentration of the suspended particles α_0, the Alfvén velocity V_A, the kinematic viscosity ν_0, and the kinematic viscoelasticity ν'_0, respectively, have stabilizing effects on the considered system, since the growth rate is decreased with increasing α_0, V_A, ν_0, or ν'_0. Note, from Fig. 8, that the mass concentration of the suspended particles α_0 has no effect on the stability of the system for wavenumbers $k \geq 10$, and that α_0 has a slightly stabilizing effect for wavenumbers $k < 10$. Figure 11 indicates that the kinematic viscoelasticity ν'_0 has no effect on the stability of the considered system for wavenumbers $k \leq 5$, while ν'_0 has a slightly stabilizing effect for wavenumbers $k > 5$. It follows also, from Figs. 12 and 13, that both the stratification parameter β and the finite resistivity η have destabilizing effects on the considered system, since the growth rate increases with the increasing β and η. In this case, for

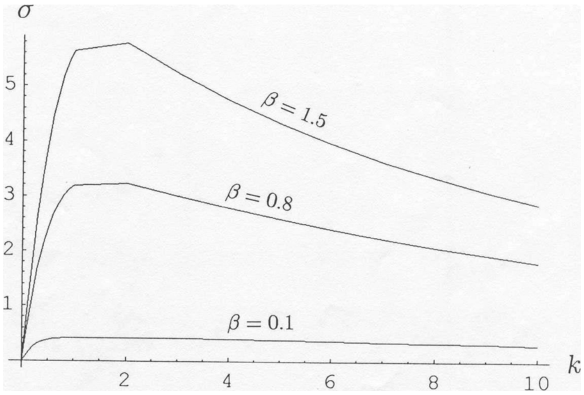

Fig. 12. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the stratification parametrt $\beta = 0.1$, 0.8, and 1.5, with the same values of the other parameters as given in Fig. 6.

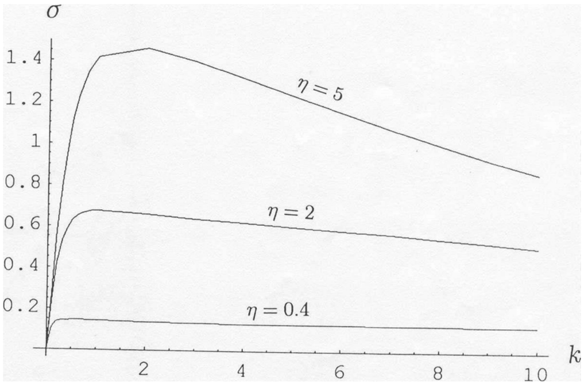

Fig. 13. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the finite resistivity $\eta = 0.4$, 2, and 5 $\mathrm{kg\,m^3\,A^{-2}\,s^{-3}}$, with the same values of the other parameters as given in Fig. 3.

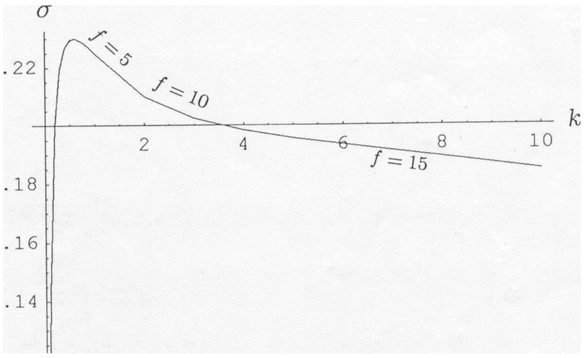

Fig. 14. The positive real part of the growth rate σ, given by (67), of the unstable mode plotted against the wavenumber k for the relaxation frequency of the suspended particles $f =$ 5, 10, and 15 $\mathrm{kg^{-1}\ m^{-1}}$, with the same values of the other parameters as given in Fig. 4.

fixed values of β and η the considered system is found to have stabilizing as well as destabilizing effects. Finally, it follows from Fig. 14 that the relaxation frequency of the suspended particles f has no effect on the stability of the considered Rivlin-Ericksen viscoelastic system for the wavenumbers range $0 \le k \le 10$, since the growth rate is constant for variabe values of $f(= 5{,}10$, and 15) at the same wavenumber; and as mentioned above, we notice that the considered system has stabilizing as well as destabilizing effects for all given constant values of f in the presence of other physical parameters.

5.2. The Case of Infinite Conductivity

In the case of infinite conductivity, i.e. when $\eta = 0$, the dispersion relation (67) takes the form

$$\sigma^3[1+v_0'L]+\sigma^2[v_0L+f(1+\alpha_0)+v_0'fL] + \sigma[v_0fL+\widetilde{\lambda}]+\widetilde{\lambda}f = 0. \tag{82}$$

It is readily seen from (82) that for $\beta < 0$, which is the criterion for stable density stratification, the above equation does not admit any positive real root, or complex root with a positive real part, implying thereby that the system is stable. For unstable density stratification ($\beta > 0$), the gas-particle medium is stable or unstable according to

$$V_\mathrm{A}^2 \gtrless (g\beta/L). \tag{83}$$

Hence it is evident from (82) and (83) that the stability criterion is independent of the presence of suspended particles, kinematic viscosity, and kinematic viscoelasticity.

From (82), we find that for $\beta > 0$ the system is unstable in the absence of a magnetic field. However, the system can be stabilized by a magnetic field which satisfies the inequality

$$V_\mathrm{A}^2 > (g\beta/L). \tag{84}$$

We find that for $\beta > 0$ and $V_\mathrm{A}^2 < (g\beta/L)$, (82) has at least one positive real root which will destabilize the medium for all wavenumbers. Let σ_0 denotes the positive real root of (82), hence

$$\sigma_0^3[1+v_0'L]+\sigma_0^2[v_0L+f(1+\alpha_0)+v_0'fL] + \sigma_0[v_0fL+\widetilde{\lambda}]+\widetilde{\lambda}f = 0. \tag{85}$$

In order to have an insight into the role of suspended particles, kinematic viscosity, and kinematic viscoelasticity on the growth rate of unstable modes, we calculate $\mathrm{d}\sigma_0/\mathrm{d}f$, $\mathrm{d}\sigma_0/\mathrm{d}\nu_0$, and $\mathrm{d}\sigma_0/\mathrm{d}\nu_0'$, and examine their nature. From (85), we obtain

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}f} = -\frac{1}{G_1}[\sigma_0^2\{(1+\alpha_0)+\nu_0'L\} + \sigma_0\nu_0 L + \widetilde{\lambda}], \tag{86}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0} = -\frac{\sigma_0 L}{G_1}(\sigma_0+f), \tag{87}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0'} = -\frac{\sigma_0^2 L}{G_1}(\sigma_0+f), \tag{88}$$

where

$$G_1 = 3\sigma_0^2[1+\nu_0'L] + 2\sigma_0[\nu_0 L + f(1+\alpha_0)+\nu_0' fL] + [\nu_0 fL+\lambda]. \tag{89}$$

Therefore, in addition to the condition $V_\mathrm{A}^2 < (g\beta/L)$, which makes the medium unstable, we find that if either

$$\widetilde{\lambda} > 3\sigma_0^2[1+\nu_0'L] + 2\sigma_0[\nu_0 L + f(1+\alpha_0)+\nu_0' fL] + \nu_0 fL \tag{90}$$

or

$$\widetilde{\lambda} < [\sigma_0^2\{(1+\alpha_0)+\nu_0'L\} + \sigma_0\nu_0 L], \tag{91}$$

$\mathrm{d}\sigma_0/\mathrm{d}f$ is always negative. In writing (90) and (91), we have taken note of the fact that $\alpha_0 (= mN/\rho)$ can not exceed 1. Thus, the growth rate decreases with increasing relaxation frequency of the suspended particles. However, the growth rate increases with increasing relaxation frequency parameter if

$$[\sigma_0^2\{(1+\alpha_0)+\nu_0'L\} + \sigma_0\nu_0 L] < \lambda < 3\sigma_0^2\{(1+\alpha_0)+\nu_0'L\} + 2\sigma_0[\nu_0 L + f(1+\alpha_0)+\nu_0' fL] + \nu_0 fL. \tag{92}$$

It is also evident, from (87) and (88) that an increase in the kinematic viscosity or kinematic viscoelasticity results in decreasing or increasing the growth rate of the disturbance if

$$\widetilde{\lambda} \lesseqgtr 3\sigma_0^2(1+\nu_0'L) + 2\sigma_0[\nu_0 L + f(1+\alpha_0)+\nu_0' fL] + \nu_0 fL, \tag{93}$$

respectively, together with the condition $V_\mathrm{A}^2 < (g\beta/L)$ in the case of stability only.

Finally, for an infinitely conducting medium ($\eta = 0$), and in the absence of suspended particles ($\tau = \alpha_0 = 0$), (34) for a Rivlin-Ericksen viscoelastic medium yields

$$\sigma^2[1+\nu_0 L] + \sigma_0\nu_0 L + \widetilde{\lambda} = 0. \tag{94}$$

Following the same procedure as above, we can deduce the growth rate with kinematic viscosity, kinematic viscoelasticity, stratification parameter, and the Alfvén velocity, respectively, in the form

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0} = -\frac{\sigma_0 L}{[2\sigma(1+\nu_0'L)+\nu_0 L]}, \tag{95}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0'} = -\frac{\sigma_0^2 L}{[2\sigma(1+\nu_0'L)+\nu_0 L]}, \tag{96}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\beta} = \frac{gk^2}{[2\sigma(1+\nu_0'L)+\nu_0 L]}, \tag{97}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}V_\mathrm{A}} = -\frac{2k^2 V_\mathrm{A}}{[2\sigma(1+\nu_0'L)+\nu_0 L]}. \tag{98}$$

From (95) and (96), we conclude that the growth rate is always negative. Comparison of (70) with (95) reveals the interesting feature that the simultaneous presence of suspended particles, viscosity, and viscoelasticity can increase as well as decrease the growth rate. If the suspended particles are not included, then both the kinematic viscosity and kinematic viscoelasticity always reduce the growth rate of the instability, and therefore they have stabilizing effects. A similar consequence of the viscosity effect has been reported by Bhatia [31], but in the context of hydromagnetic Rayleigh-Taylor instability of two superposed fluids. Also, comparison of (73) and (74) with (97) and (98), respectively, reveals that the simultaneous presence of suspended particles and density stratification (or magnetic field) can increase as well as decrease the growth rate, whereas, if suspended particles are absent, the stratification parameter always increases the growth rate of the instability, and therefore it has a destabilizing influence on the system, while the magnetic field always decreases the growth rate of the instability.

6. Viscid Finitely Conducting Dusty Plasma

In the limiting case of absence of kinematic viscoelasticity, i.e. when ν_0', (36) reduces to

$$\begin{aligned}\sigma^4 &+ \sigma^3[L(\nu_0+\eta)+f(1+\alpha_0)] \\ &+ \sigma^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\nu_0\eta L^2+\widetilde{\lambda}] \\ &+ \sigma[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda}f]-gk^2\beta\eta f=0.\end{aligned} \tag{99}$$

Note that, in this limiting case, (99) differs from the corresponding dispersion relation obtained by Sanghvi and Chhajlani [39] in two terms, the constant term and the coefficient of the σ term, due to an error in the algebra (cf. (31)–(33) in their work), and hence their dispersion relation is incorrect. Here, we shall reconsider their stability discussion in the light of the correct dispersion relation given by (99). We find that for $\beta<0$, a stable stratification remains stable, since then (99) will not allow any positive real root, or complex root with positive real part. For $\beta>0$ there is at least one positive real root, leading to instability of the considered system. We denote this root by σ_0, which should satisfy the equation

$$\begin{aligned}\sigma_0^4 &+ \sigma_0^3[L(\nu_0+\eta)+f(1+\alpha_0)] \\ &+ \sigma_0^2[fL\{\nu_0+\eta(1+\alpha_0)\}+\nu_0\eta L^2+\widetilde{\lambda}] \\ &+ \sigma_0[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda}f]-gk^2\beta\eta f=0.\end{aligned} \tag{100}$$

In order to determine the roles of the suspended particles, kinematic viscosity, and the finite resistivity in an explicit manner, we calculate $\mathrm{d}\sigma_0/\mathrm{d}f$, $\mathrm{d}\sigma_0/\mathrm{d}\nu_0$, and $\mathrm{d}\sigma_0/\mathrm{d}\eta$ from (100), which yields

$$\begin{aligned}\frac{\mathrm{d}\sigma_0}{\mathrm{d}f} = -\frac{1}{K_1}&[\sigma_0^3(1+\alpha_0)+\sigma_0^2L\{\nu_0+\eta(1+\alpha_0)\} \\ &+\sigma_0(\nu_0\eta L^2+\widetilde{\lambda})-gk^2\beta\eta],\end{aligned} \tag{101}$$

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\nu_0} = -\frac{\sigma_0 L}{K_1}(\sigma_0+f)(\sigma_0+\eta L), \tag{102}$$

$$\begin{aligned}\frac{\mathrm{d}\sigma_0}{\mathrm{d}\eta} = -\frac{L}{K_1}&[\sigma_0^3+\sigma_0^2\{f(1+\alpha_0)+\nu_0 L\} \\ &+\sigma_0\{\nu_0 fL-(gk^2\beta/L)\}-f(gk^2\beta/L)]\end{aligned} \tag{103}$$

where

$$\begin{aligned}K_1 = 4\sigma_0^3&+3\sigma_0^2[L(\nu_0+\eta)+f(1+\alpha_0)] \\ &+2\sigma_0[fL\{\nu_0+\eta(1+\alpha_0)\}+\nu_0\eta L^2+\widetilde{\lambda}] \\ &+[\eta(\nu_0 fL^2-gk^2\beta)+\widetilde{\lambda}f].\end{aligned} \tag{104}$$

Now consider the inequalities

$$\begin{aligned}&(\sigma_0+\eta L)(gk^2\beta/L) \\ &\quad\lesseqgtr [\sigma_0^3\{(1+\alpha_0)+\sigma_0^2 L\{\nu_0+\eta(1+\alpha_0)\} \\ &\qquad+\sigma_0\{\nu_0\eta L^2+k^2V_\mathrm{A}^2]\end{aligned} \tag{105}$$

$$\begin{aligned}&2\sigma_0(gk^2\beta/L) \\ &\lesseqgtr [4\sigma_0^3+3\sigma_0^2\{L(\nu_0+\eta)+f(1+\alpha_0)\} \\ &\quad+2\sigma_0\{fL(\nu_0+\eta(1+\alpha_0))+\eta L^2\nu_0+k^2V_\mathrm{A}^2\} \\ &\quad+\{\eta\nu_0 fL^2+fk^2V_\mathrm{A}^2\}]\end{aligned} \tag{106}$$

$$\begin{aligned}&(gk^2\beta/L)(\sigma_0+f) \\ &\quad\lesseqgtr \sigma_0[\sigma_0^2+\sigma_0\{f(1+\alpha_0)+\nu_0 L\}+\nu_0 fL]\end{aligned} \tag{107}$$

If the upper (or lower) signs of the inequalities (105) and (106) are satisfied simultaneously, we find that $\mathrm{d}\sigma_0/\mathrm{d}f$ is negative, and if the upper and lower signs of these inequalities, or vice-versa hold, respectively, then $\mathrm{d}\sigma_0/\mathrm{d}f$ turns out to be positive. Also, if the upper (or lower) signs of the inequalities (106) and (107) are satisfied simultaneously, we find that $\mathrm{d}\sigma_0/\mathrm{d}\eta$ is negative, and if the upper and lower signs of the inequalities (106) and (107), or vice-versa hold, respectively, then $\mathrm{d}\sigma_0/\mathrm{d}\eta$ is positive. Finally, $\mathrm{d}\sigma_0/\mathrm{d}\nu_0$ turns out to be negative or positive if the inequality (106) holds for the upper and lower signs, respectively. Thus, we conclude that the suspended particles, kinematic viscosity, and the finite resistivity have stabilizing as well as destabilizing effects on the considered system according to the above mentioned limitations expressed in therms of the other physical parameters even in the absence of kinematic viscoelasticity of the medium. It is also readily seen from (99) that for an infinite plasma ($\eta=0$) all the coefficients of (99) will be positive if $\widetilde{\lambda}>0$, i.e. when the condition $V_A^2>(g\beta/L)$ is satisfied, as mentioned above, leading thereby to the stability of the considered system.

For a dusty inviscid finitely conducting medium we must deduce the dispersion relation corresponding to an inviscid plasma ($\nu_0=0$), We find that the dispersion relation (99) reduces to

$$\begin{aligned}\sigma^4 &+ \sigma^3[\eta L+f(1+\alpha_0)] \\ &+ \sigma^2[fL\eta(1+\alpha_0)+\widetilde{\lambda}] \\ &+ \sigma[\widetilde{\lambda}f-gk^2\beta\eta]-gk^2\beta\eta f=0.\end{aligned} \tag{108}$$

In the derivation of the above equation, the effect of the mass (m) and size (r) of the suspended particles has been included in the analysis through the quantity f. For $\beta < 0$ the above equation will not admit any real positive root, or complex root with real positive part, and so the system remains stable. For $\beta > 0$, the above equation will possess at least one real positive root which destabilizes the system for all wavenumbers. It is noteworthy from (108) that for an infinitely conducting plasma ($\eta = 0$), all the coefficients of (108) are positive when $\widetilde{\lambda} > 0$, i.e. for the wavenumbers range given by the inequality $k^2 < [(g\beta/V_\mathrm{A}^2) - (\widetilde{m}\pi/d)^2]$, which means that the considered system is stable in the absence of both the kinematic viscosity and kinematic viscoelasticity, as indicated recently by Al-Khateeb and Leham [49], where they have found that the magnetic field acts as a stabilizer up to a threshold value that can be determined from the obtained dispersion relation. Therefore, the applied magnetic field stabilizing in this case, and the stability of the system holds for values of the Alfvén velocity greater than the critical value $V_\mathrm{A}^2 = g\beta/L$.

7. Summary and Concluding Remarks

In conclusion, we have examined the implication of simultaneous inclusion of both finite resistivity and the suspended particles effect on the Rayleigh-Taylor instability of two models of a magnetized and viscoelastic (Walters B' or Rivlin-Ericksen) medium having a vertical density stratification. The horizontal magnetic field, and the viscosity as well as the viscoelasticity of the medium are assumed to be variable. The following concluding remarks have been outlined:

1) Firstly, for the case of the Walters B' viscoelastic fluid model, we have found, for both the longitudinal and transverse modes, that for stable density stratification, the system can be stabilized under certain conditions depending on all physical quantities included in the analysis, whereas the unstable density stratification remains unstable for all wavenumbers which can be stabilized by a suitable choice of the magnetic field for vanishing resistivity. Thus, the finite resistivity is found to have a destabilizing influence on the Rayleigh-Taylor configuration. The variations of the growth rate of the unstable Rayleigh-Taylor modes with the relaxation frequency of the suspended particles, kinematic viscosity, kinematic viscoelasticity, finite resistivity, magnetic field, and the stratification parameter have been evaluated analytically. All the physical parameters are found to have stabilizing as well as destabilizing effects on the considered system under certain conditions depending on the other parameters. The case of a dusty plasma with infinite conductivity and absence of a magnetic field is also considered, and the stability conditions are obtained, from which we conclude that the stable stratification remains stable only for a wavenumber range depending on the kinematic viscoelasticity, while the unstable stratification remains unstable irrespective whether the medium is viscoelastic or not. It is found also that the presence of dust always reduces the growth rate of the unstable Rayleigh-Taylor perturbations in this case. The numerical results for the Walters B' viscoelastic fluid model can be summarized as follows:

(i) Both the mass concentration of the suspended particles α_0, and the kinematic viscosity ν_0 have destabilizing effects on the considered system. For fixed small values of α_0 and ν_0, the system is found to have stabilizing as well as destabilizing effects. It is found also that α_0 has no effect on the stability of the system for wavenumbers $k \geq 10$.

(ii) The finite resistivity η has a stabilizing effect, while the relaxation frequency of the suspended particles f has a destabilizing influence on the system. For a constant value of f, the considered system is found to have a stabilizing influence in this case, and f is found to have no effect on the stability of the system for the wavenumbers $k \geq 5$.

(iii) The Alfvén velocity V_A has a destabilizing effect, while the stratification parameter β has a stabilizing influence on the system. It is found also that the system is stabilized for small constant Alfvén velocities, while for constant large Alfvén velocities, the system is found to be stable as well as unstable. For small stratification parameters, the system is found to be unstable, while for larger stratification parameters $\beta \geq 1.5$, then the system is found to be always stable.

(iv) The kinematic viscoelasticity has a stabilizing effect on the considered system.

2) Secondly, for the case of Rivlin-Ericksen viscoelastic fluid it is found that a stable (or unstable) density stratifications will remain stable (or unstable), respectively, even in the presence of the considered parameters. The behaviour of the growth rate of the unstable mode with respect to the relaxation frequency of the suspended particles, kinematic viscosity, kinematic viscoelasticity, finite resistivity, and the stratification parameters, is examined. The above mentioned parameters are found to have stabilizing as well as desta-

bilizing effects on the considered system in the presence of finite resistivity under certain conditions. For an infinitely conducting medium it is found that both the relaxation frequency of the suspended particles and the kinematic viscosity have a stabilizing effect, while both the kinematic viscoelasticity and stratification parameter have a destabilizing influence on the considered system if the condition $V_A^2 > (g\beta/L)$ is satisfied. The variation of the growth rate of the unstable mode with respect to the relaxation frequency of the suspended particles, kinematic viscosity, and kinematic viscoelasticity are calculated, and the stability conditions in each case are obtained and discussed in detail. Finally, the limiting case of vanishing finite resistivity and suspended particles is examined. It was found that both the kinematic viscosity and kinematic viscoelasticity have a stabilizing effect, while the stratification parameter is found to have a destabilizing effect in this case, even in the presence of the other parameters. The numerical results for the Rivlin-Ericksen viscoelastic fluid model can be summarized as follows:

(i) The mass concentration of the suspended particles α_0, and the Alfvén velocity V_A, the kinematic viscosity ν_0, and the kinematic viscoelasticity ν_0' have stabilizing effects on the considered system. It is found also that α_0 has no effect on the stability of the system for wavenumbers values $k \geq 10$, and that α_0 has a slightly stabilizing effect for wavenumber range $0 < k < 10$. The kinematic viscoelasticity ν_0' has no effect on the stability of the system for wavenumbers $k \leq 5$, while it has a slightly destabilizing influence on the system for wavenumbers $k > 5$.

(ii) Both the stratification parameter β and the finite resistivity η have destabilizing effects on the considered system.

(iii) The relaxation frequency of the suspended particles f has no effect on the stability of the considered system.

3) The case of vanishing kinematic viscoelasticity is examined, i. e. the limiting case of a viscid (and inviscid) finitely conducting dusty plasma is considered, and the corrected dispersion relation is obtained, from which the stability analysis in this case in discussed. It is found that the magnetic field has a stabilizing effect in the absence of both viscosity and finite resistivity, and stability of the system occurs for values of the Alfvén velocity satisfying the inequality $V_A^2 > (g\beta/L)$. Thus the kinematic viscosity always suppresses the growth rate of the unstable Rayleigh-Taylor perturbations for an ideal plasma. On the other hand it enhances the growth rate when the medium is considered to be finitely conducting. The limiting case of absence of finite resistivity is also considered, and it is found that the stable density stratification remains stable even in the presence of the remaining parameters, while for the unstable density stratification it follows that the gas-particle medium is stable or unstable according as $V_A^2 \gtrless (g\beta/L)$, respectively. This condition is independent of the presence of suspended particles, kinematic viscosity, and kinematic viscoelasticity.

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[1] S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Dover Publications, New York 1981.
[2] R. G. Larson, Rheol. Acta **31**, 213 (1992).
[3] R. B. Bird, R. C. Armstrong, and H. Hassager, Dynamics of Polymeric Liquids, Vol. 1. Fluid Mechanics, John Wiley & Sons, New York 1987.
[4] D. D. Joseph, Fluid Dynamics of Viscoelastic Fluids, Applied Mathematical Sciences, Vol. 28, Springer-Verlag, New York 1990.
[5] K. Walters, in Second-Order Effects in Elasticity, Plasticity and Fluid Dynamics, Pergamon Press, Oxford 1964.
[6] D. Beard and K. Walters, Proc. Camb. Phil. Soc. **60**, 667 (1964).
[7] A. Raptis and C. P. Perdilis, Int. J. Engng. Sci. **23**, 51 (1985).
[8] A. Raptis, G. Tzivanidis, and N. Kafousios, Lett. Heat Mass Transfer **8**, 417 (1981).
[9] A. Raptis, N. Kafousios, and C. Massulas, Appl. Math. Mech. (ZAMM) **62**, 489 (1982).
[10] G. Chakraborty and P. R. Sengupts, Proc. Nat. Acad. Sci. India **64**, 75 (1994).
[11] R. P. Sharma and H. Kumar, Indian J. Pure Appl. Math. **25**, 331 (1994); Proc. Nat. Acad. Sci. India **65**, 75 (1995).
[12] M. I. A. Othman, Int. J. Engng. Sci. 39, 1217 (2001).
[13] M. F. El-Sayed, Arch. Appl. Mech. **71**, 717 (2001), Eur. Phys. J. D **23**, 1137 (2003).

[14] M. F. El-Sayed, Chaos, Solitons & Fractals **14**, 1137 (2002); Eur. Phys. J. B **37**, 241 (2004).

[15] M. F. El-Sayed, Z. Naturforsch. **55a**, 460 (2000); **56**, 416 (2001).

[16] R. C. Sharma and P. Kumar, Czech. J. Phys. **47**, 197 (1997); Z. Naturforsch. **52a**, 528 (1997).

[17] R. C. Sharma, Sunil, and S. Chand, Czech. J. Phys. **49**, 189 (1999); Indian J. Pure Appl. Math. **31**, 49 (2000).

[18] R. C. Sharma and S. K. Kango, Czech. J. Phys. **49**, 197 (1999).

[19] P. Kumar, Z. Naturforsch. **54a**, 343 (1999); Indian J. Pure Appl. Math. **30**, 1117 (1999); Z. Angew. Math. Phys. (ZAMP) **51**, 912 (2000).

[20] S. Özer and E. S. Suhubi, ARI **51**, 221 (1999).

[21] S. Benkadda and V. N. Tsytovich, Phys. Plasmas **2**, 2970 (1995).

[22] R. K. Chhajlani and A. K. Parihar, Astrophys. J. **422**, 746 (1994).

[23] R. C. Sharma, K. Prakash, and S. N. Dube, Acta Phys. Acad. Sci. Hungaricae **40**, 3 (1976).

[24] R. C. Sharma and K. C. Sharma, Acta Phys. Acad. Sci. Hungaricae **43**, 251 (1977).

[25] R. K. Chhajlani, R. K.Sanghvi, and P. Purohit, Z. Naturforsch. **39a**, 939 (1984).

[26] P. K. Shukla and H. U. Rahman, Phys. Plasmas **3**, 430 (1996).

[27] G. T. Birk, A. Kopp, and P. K. Shukla, Phys. Plasmas **3**, 3564 (1996).

[28] G. T. Birk, Phys. Plasmas **9**, L745 (2002).

[29] B. A. Maury and R. Glowinski, C. R. Acad. Sci. Paris (Ser. I) **324**, 1079 (1997).

[30] F. Verheest, Waves in Dusty Space Plasmas, Kluwer, Dordrecht 2000.

[31] M.-B. Kallenrode, Space Physics: Plasmas and Particles in the Heliosphere and Magnetosphere, Springer-Verlag, Berlin 1998.

[32] P. J. Sturrock, Plasma Physica: An Introduction to the Theory of Astrophysical, Geophysical, and Laboratory Plasmas, Cambridge University Press 1994.

[33] R. J. Goldston and P. H. Rutherford, Intorduction to Plasma Physics, Institute of Physics, Bristol 1995.

[34] N. Seehafer and J. Schumacher, Phys. Plasmas **5**, 2363 (1998).

[35] H. P. Furth, J. Killen, and M. N. Rosenbluth, Phys. Fluids **6**, 453 (1963).

[36] J. D. Jukes, J. Fluid Mech. **16**, 187 (1963).

[37] L. N. Zadoff and M. Begun, Phys. Fluids **11**, 1238 (1968).

[38] A. K. Sundram, Phys. Fluids **11**, 1709 (1968).

[39] R. K. Sanghvi and R. L. Chhajlani, Z. Naturforsch. **40a**, 826 (1985).

[40] C. Lamb, Hydromagnetic instability in the earth's core, Ph. D. Thesis, University of Glasgow 1994.

[41] D. K. Callebaut, M. F. El-Sayed, and N. L. Tsintsadze, Ukr. J. Phys. **40**, 472 (1995).

[42] M. F. El-Sayed, Phys. Rev. E **60**, 7588 (1999); Czech. J. Phys. **49**, 473 (1999)

[43] R. K. Chhajlani and M. K. Vyas, Astrophys. Space Sci. **173**, 109 (1990).

[44] M. F. El-Sayed, Nuovo Cim. D **20**, 1645 (1998); Czech. J. Phys. **50**, 407; 607 (2000).

[45] P. K. Bhatia, Nuovo Cim. B **19**, 161 (1974).

[46] E. A. Spiegel, Astrophys. J. **141**, 1068 (1965).

[47] D. H. Michael, Proc. Camb. Phil. Soc. **61**, 569 (1965).

[48] L. Xu, Commun. Nonlinear Sci. Num. Simul. **7**, 165 (2002).

[49] A. M. Al-Khateeb and N. M. Leham, Contrib. Plasma Phys. **43**, 25 (2003).